RWTH Aachen **SS 2012**
Lehrgebiet Theoretische Informatik Problem Set 6
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Analysis of Algorithms — Tutorial

Problem 6-1

Use the operator technique to obtain a solution to the following recurrence:

$$a_{n+2} = (n+2)a_{n+1} - na_n + n,$$

with $a_0 = 0 = a_1$.

Problem 6-2

Let a_n be the number of calls to $f(n,k)$ in the following program.

```
int f(n,k) {
    int sum = k;
    if(n > 1){
        for(int i = 0; i ≤ n; i++){
            sum += f(n − 1, i);
            sum += f(n − 1, 2 ∗ i);
            sum += f(n − 1, 3 ∗ i);
            for(int j = 0; j < n; j++){
                sum += f(n − 2, i + j);
                sum += f(n − 2, i + 2 ∗ j);
            }
        }
    } else {
        sum = n;
    }
    return sum;
}
```

1. Determine the recurrence of a_n and express it in terms of the shift operator E.
2. Reduce the degree of the recurrence by factorizing the operator expression. Solve the newly obtained recurrence b_n.

Homework Assignment 6-1 (10 Points)

Provide a closed form for a_n from Problem 6-2 (up to initial conditions), using the solution obtained for b_n in class[1].

[1] $b_{n-2} = 2^{n-3}n!$

Homework Assignment 6-2 (10 Points)

Solve the following recurrence by reducing its degree.

$$\begin{aligned} a_0 &= 8000 \\ a_1 &= \frac{1}{2} \\ a_{n+2} + a_{n+1} - n^2 a_n &= n! \end{aligned}$$